MONASH University

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Forecast reconciliation with linear combinations

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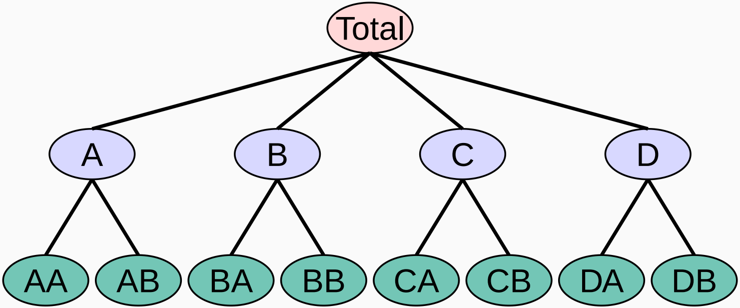
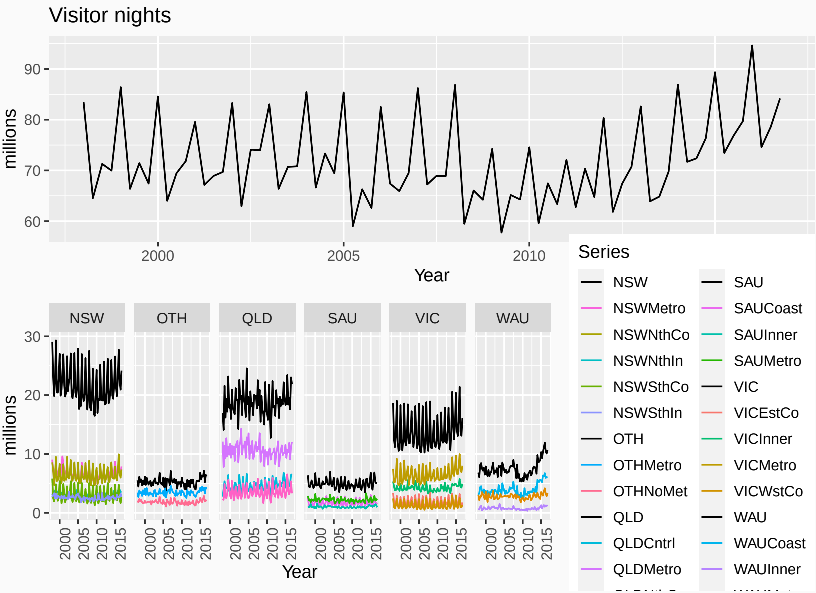
Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.



Australian tourism data



Research question

How to improve **forecast accuracy** of **coherent** forecasts?

Coherence

The forecasts can add up in a manner that is consistent with the aggregation structure of the collection of time series.

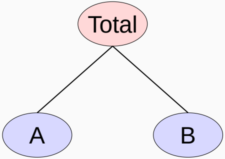
Reconciliation

Reconciliation is a method to achieve coherence

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{SG}\hat{\boldsymbol{y}}_h,$$

- $\tilde{\boldsymbol{y}}_h$ = vector of h-step-ahead coherent forecasts.
- $\boldsymbol{S}$ = summing matrix containing the linear constraints.
- $\hat{\boldsymbol{y}}_h$ = vector of h-step-ahead forecasts of all the series.
- $\boldsymbol{G}$ = matrix mapping the forecasts for all levels into the bottom-level:

Reconciliation



$$\tilde{\boldsymbol{y}}_h = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_h$$

$$\underbrace{\begin{bmatrix} \tilde{y}_h \\ \tilde{y}_{A,h} \\ \tilde{y}_{B,h} \end{bmatrix}}_{\tilde{\boldsymbol{y}}_h} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{S}} \boldsymbol{G} \underbrace{\begin{bmatrix} \hat{y}_{Total,h} \\ \hat{y}_{A,h} \\ \hat{y}_{B,h} \end{bmatrix}}_{\hat{\boldsymbol{y}}_h}$$

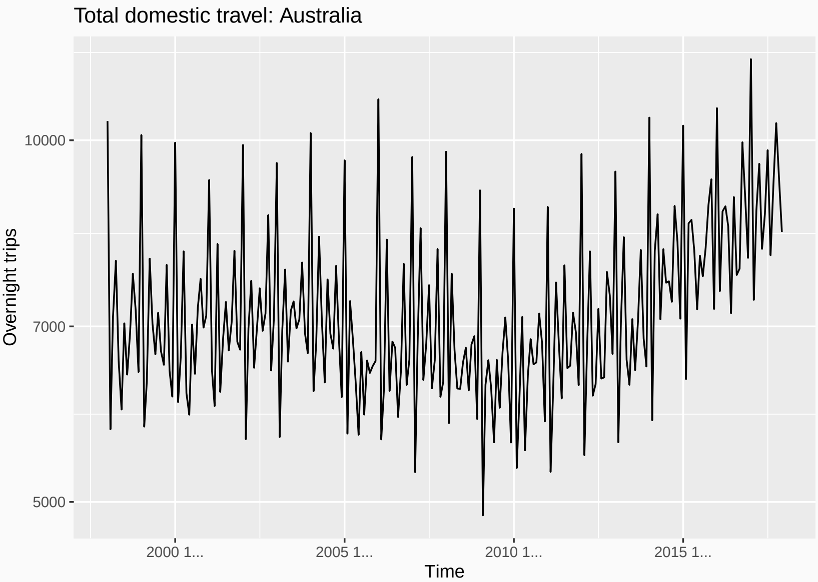
Forecast reconsiliation

1. Forecast series at all levels using any forecast technique (ETS, ARIMA, etc).
2. Reconcile the forecasts by **finding the optimal *G*** so they are coherent.

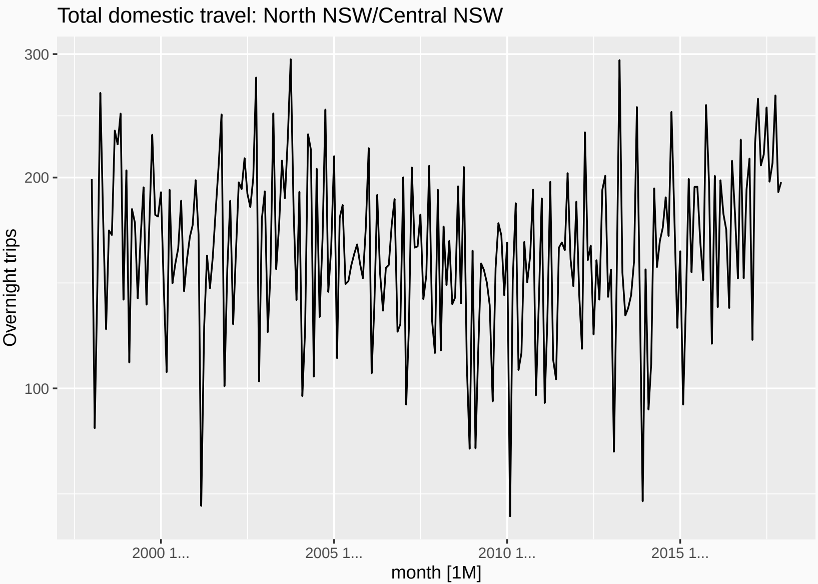
Key references

- Athanasopoulos, Ahmed, and Hyndman (2009) Hierarchical Forecasts for Australian Domestic Tourism.
- Hyndman et al. (2011) Optimal Combination Forecasts for Hierarchical Time Series.
- Wickramasuriya, Athanasopoulos, and Hyndman (2019) Optimal Forecast Reconciliation for Hierarchical and Grouped Time Series Through Trace Minimization.
- Panagiotelis et al. (2021) Forecast Reconciliation: A Geometric View with New Insights on Bias Correction.

Australian tourism data



Australian tourism data



Observation

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Better signal-noise ratio in the aggregated data

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What is aggregation?

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The aggregations are just **linear combinations**!

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Better signal-noise ratio in the aggregated data

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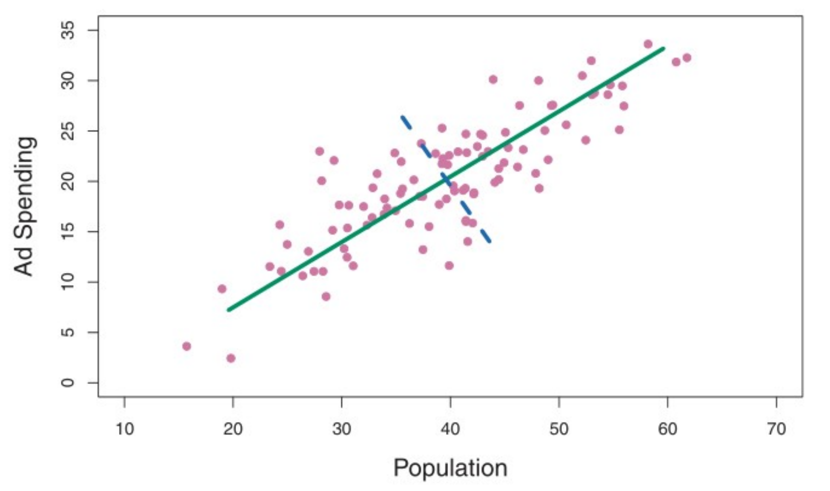
The aggregations are just **linear combinations**!

Research Question

Finding the linear combination that can be best forecasted.

Principal Component Analysis (PCA)



James et al. (2014)

Forecastable Component (ForeC)

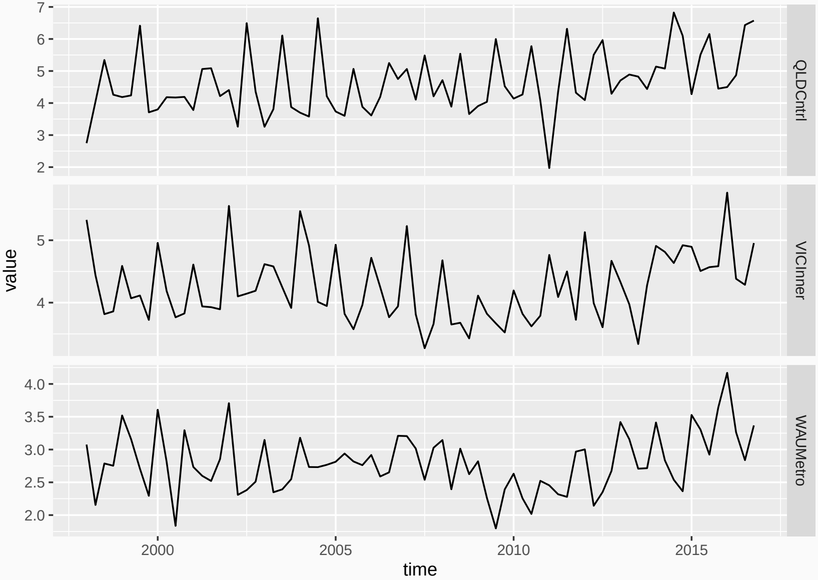
Forecastable component (Goerg 2013) maximise **forecastability**, finding linear combinations with **most regular patterns**.

Forecastability

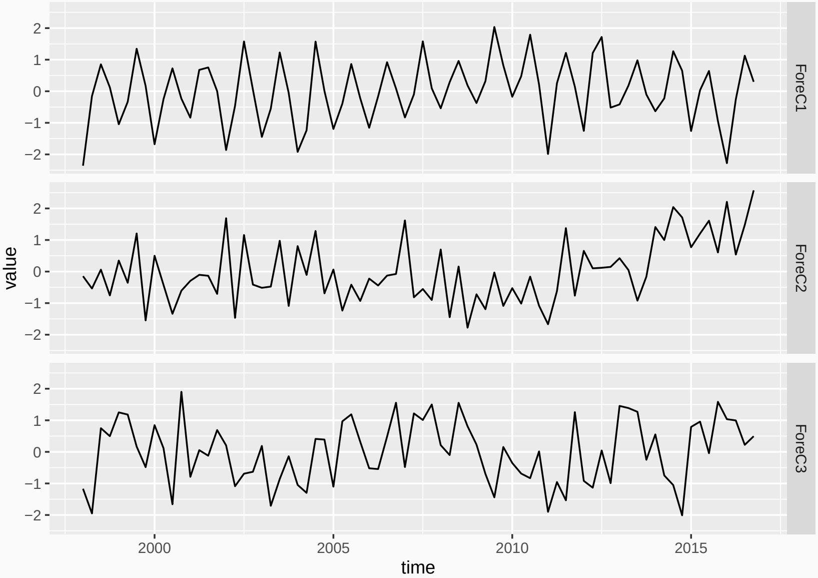
$$\Omega\left(y_t\right) = 1 - H\left(y_t\right),$$

where $H(y_t)$ is The Shannon entropy (Shannon 1948) of spectral density of y_t

Example: Australian tourism



Example: Australian tourism



Forecast

Taking the first r components (i.e. linear combination)

$$\underset{T\times k}{\mathbf{Y}}\underset{k\times r}{\mathbf{W}_{(r)}} = \underset{T\times r}{\mathbf{C}_{(r)}},$$

where $\mathbf{C}_{(r)}$ is the first r components, $\mathbf{W}_{(r)}$ is the weighting matrix.

Forecast

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where $\mathbf{C}_{(r)}$ is the first r components, $\mathbf{W}_{(r)}$ is the weighting matrix.

The forecast of $\mathbf{Y}$ can be made from forecast of $\mathbf{C}_{(r)}$ obtained using any method:

$$\underset{h\times k}{\widehat{\mathbf{Y}}} = \underset{h\times r}{\widehat{\mathbf{C}}_{(r)}}\,\underset{r\times k}{\widehat{\mathbf{V}}^{T}_{(r)}},$$

where $\mathbf{V}^{T}_{(r)}$ transforms components back.

Results

To be continued..

References i

Athanasopoulos, George, Roman A Ahmed, and Rob J Hyndman. 2009. “Hierarchical Forecasts for Australian Domestic Tourism.” *Int. J. Forecast.* 25 (1): 146–66. `https://doi.org/10.1016/j.ijforecast.2008.07.004`.

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Hyndman, Rob J, Roman A Ahmed, George Athanasopoulos, and Han Lin Shang. 2011. “Optimal Combination Forecasts for Hierarchical Time Series.” *Comput. Stat. Data Anal.* 55 (9): 2579–89. `https://doi.org/10.1016/j.csda.2011.03.006`.

James, Gareth, Daniela Witten, Trevor Hastie, and Robert Tibshirani. 2014. *An Introduction to Statistical Learning: With Applications in R*. Springer New York. `https://doi.org/10.1007/978-1-4614-7138-7`.

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Shannon, C E. 1948. "A Mathematical Theory of Communication." *The Bell System Technical Journal* 27 (3): 379–423. `https://doi.org/10.1002/j.1538-7305.1948.tb01338.x`.

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